Math 155, ***Lecture Notes- Bonds*** **Name____________**

Section 8.2 *Integration by Parts*

Recall that the product rule for differentiation states that

$$\frac{d}{dx}\big(u(x)v(x)\big)=u(x)\frac{d}{dx}\big(v(x)\big)+v(x)\frac{d}{dx}\big(u(x)\big)$$

or

$$u(x)\frac{d}{dx}\big(v(x)\big)=\frac{d}{dx}\big(u(x)v(x)\big)-v(x)\frac{d}{dx}\big(u(x)\big)$$

Integration on both sides of this equation gives the following:

$$\int u(x)v'(x)dx=u(x)v(x)-\int v(x)u'(x)dx$$

Rewriting with *dv* and *du* differentials gives us

$$\int udv = uv - \int vdu$$

THEOREM 8.1 Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du.$$

Ex.1 Integrate: $\int x\sin(x)dx$

$$\int udv = uv - \int vdu$$

Ex.2 Evaluate: $\int \ln(3x)dx$

$$\int udv = uv - \int vdu$$

Ex.3 Evaluate: $\int x^2\cos(x)dx$

$$\int udv = uv - \int vdu$$

Ex.4 Evaluate: $\int_0^1 x \arcsin\left(x^2\right) dx$

$$\int u dv = uv - \int v du$$

Ex.5 Evaluate: $\int x^2 e^{2x} dx$

$$\int u\,dv = uv - \int v\,du$$

Ex.6 Evaluate: $\int e^{x}\cos(2x)\,dx$

$$\int u\,dv = uv - \int v\,du$$

Ex.7 Evaluate: $\displaystyle\int_0^{\frac{\pi}{4}} x \sec^2(x)\,dx$

Tabular Method

$$\int u\,dv = uv - \int v\,du$$

Ex.8 Evaluate: $\int x^3 e^{-2x} dx$

Guidelines for Integration by Parts

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.

2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Summary of Common Integrals Using Integration by Parts

Summary of Common Integrals Using Integration by Parts

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \qquad \int x^n \sin ax\,dx, \qquad \text{or} \qquad \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}\,dx$, $\sin ax\,dx$, or $\cos ax\,dx$.

2. For integrals of the form

$$\int x^n \ln x\,dx, \qquad \int x^n \arcsin ax\,dx, \qquad \text{or} \qquad \int x^n \arctan ax\,dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\,dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \qquad \text{or} \qquad \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\,dx$.